MECHANICAL ENGINEERING DEPARTMENT

Design of a Gear Box

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**Table of Content:**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Theoretical Principles</td>
<td>3</td>
</tr>
<tr>
<td>Calculations</td>
<td>9</td>
</tr>
<tr>
<td>Results/ Summary</td>
<td>22</td>
</tr>
<tr>
<td>Discussion</td>
<td>27</td>
</tr>
<tr>
<td>Conclusions</td>
<td>47</td>
</tr>
<tr>
<td>References</td>
<td>48</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>49</td>
</tr>
<tr>
<td>Appendix</td>
<td>50</td>
</tr>
</tbody>
</table>
Abstract:

Bearing designs have changed significantly over the last 50 years and so it is a common practice to replace the older bearings in machines with better designed and more efficient bearings. This will result in a more efficient machine with the ability to have less power losses, better stability of the shaft, and or rotate at a higher speed. Another reason changing bearings in a machine is desired because changing the power source to one that operates at a higher speed may provide the ability to reduce fuel consumption. Given the initial conditions of a transmission used to reduce the speed of the input shaft, we have been given the opportunity to redesign a transmission so that its input shaft can withstand a higher rotational velocity as well as to upgrade the bearings in the transmission so that the shafts can rotate at a higher speed and the whole transmission can operate more efficiently.

This includes selecting new bearings that can meet the conditions that the transmission will impart onto them, like higher loads and rotational speeds. Some characteristics of the transmission had to be assumed or based off of ratios on the schematic of the original transmission. To meet the application of the higher speed and new bearings, the transmission’s geometric features had to be reconsidered with slight modifications so that this transmission runs more efficiently.
**Introduction:**

This project involves taking a given gearbox with its given conditions and dimensions and optimizing it for a new set of input conditions. A gearbox (also called a transmission) is used to transmit power from one source to another. When used, a gearbox will reduce the speed of the output compared to the input and increase the torque of the output compared to the input.

A gearbox is usually used when there is a need for a change in direction of the torque being applied. This can be used when the input and output shafts are not coaxial. The design of the gearbox heavily depends on the selection of a bearing to withstand the forces applied from the gears. Without a bearing to support the correct force, the shaft can not be manufactured nor the gear. Once the bearing is selected to withstand the forces, the shaft diameters can be chosen and the gear bore diameters as well. The gearbox can be designed in several ways, for example double reduction, multi reduction. The selection of a double reduction and multi reduction speed ratio gearbox depends the speed ratio that is desired. For the purpose of this project, a double reduction gearbox will be taken into consideration. An illustration of a multi reduction step gear box is shown in the following figure.

![Illustration of a multi reduction step gear box](image)

Figure: An illustration of a multi reduction step gear box used in Automobile.
There is another component in the shaft design and that is creating a keyway and key that will transmit the torque between the shaft and gear. This is crucial as well because without the correct key, the torque would not be transmitted and the gear could slide off the shaft. In the design of this project, the factor of safety was made high enough, at least a factor of safety of 3, that the design could be manipulated when need without compromising safety and efficiency. This can help the design very much as it in this case. The shafts original designs were changed to match standard bearing sizes, thus causing the bore size of the gear to have to be changed. With these major changed to the design, the gearbox is still operating well and safely according to the theoretical calculations.

Gear box finds its application in Agitators, conveyors, crushers, cranes, elevators, feeders, small ball mills, mixers, cooling towers, Extruders, Packaging and Filters. Some of the industries that make use of the gearbox are Elevator, Cement, Paper, Textile, Solvent Extraction, Plastic & leather, Rubber, Steel industry, Power plants, Mines and minerals, Waste water treatment and many other industries.
**Theoretical Principles:**

*Design of shaft:*

The design of the shaft was based on the principle of Von-Mises theory of failure (Distortion energy theory). This theory was used in order to practice non-conservative approach in shaft design, it saves some material to be used for the shaft. Some of the assumptions made for shaft design and the known parameters are defined as follows:

**Known parameters:**

Input Power: \( \dot{E} = 2000 \text{ KW} \)

Input Speed: \( N = 6000 \text{ RPM} \)

**Assumptions:**

1) The efficiency of power transmission from one shaft to the other is 100%. In simple words, the power \( \dot{E} \) remains the same throughout the transmission process.

2) The shaft would fail by a combination of static and fatigue loading only; however, a factor of safety \( F_s = 1.5 \) is taken to take into account the fatigue loading condition.

3) The critical key way factor of shaft is taken to be \( k = 1.6 \) for high loading condition on a medium carbon steel.

4) The gear diameter is user-defined taking into consideration a number of iterates before reaching to an optimized pitch diameter for the gear. The optimized gear diameter on shaft 1 is 300 mm.

**Gear Radial and Tangential force theory:**

The tangential force and the radial force acting on the gear tooth is given as: (Note: \( \phi = \) Pressure angle of the gear typically \( 20^\circ \))

**Gear Design and transmission of loads:**

For the double speed reduction, a simple spur gears are utilized. The meshing and load calculations of spur gears is relatively simple. The Gear force calculations can be performed using the following methodology:

\[
F_r = \frac{\dot{E}}{\omega R}
\]

\[
F_r = F \tan \phi
\]

\[
\omega = \frac{2\pi N}{60}
\]

The Gear meshing properties and speed reduction ratios can be shown with the help of the figure shown below. The Theory was obtained from *Design of Machine Elements, 8th Edition*, Chapter 10 Spur Gears and Chapter 3, Shaft Design.
In general, it can be inferred that:

\[ \omega_{in} r_{in} = \omega_{out} r_{out} \quad (4b) \]

\[ T_{out} r_{in} = T_{in} r_{out} \quad (4c) \]

The radial force acting on the shaft is given as:

\[ F_{tr} = \sqrt{F_t^2 + F_r^2} \quad (5) \]

The torque acting on the shaft as a result of the power transmission is given as follows:

\[ T = \frac{\dot{E}}{\omega} \quad (6) \]

The shaft loading condition can now be shown pictorially as follows:
The shaft (Equivalent beam) diagram can be solved to obtain the maximum bending moment and shear force diagrams to find the reaction forces acting on the bearings at both the ends of the shaft.

The shear stress acting on the shaft can be shown as:

\[ \tau = \frac{Tr}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{4}{3} \pi \left(\frac{d}{2}\right)^3} = \frac{16T}{\pi d^3} \]  

(7a)

From the bending moment diagram, the maximum bending moment can be utilized to calculate the maximum bending stress acting on the shaft. Let the maximum bending moment be \( M \). The maximum bending stress can be given as follows:

\[ \sigma_b = \frac{Mc}{J} = \frac{M\left(\frac{d}{2}\right)}{\frac{4}{3} \pi \left(\frac{d}{2}\right)^3} = \frac{32M}{\pi d^3} \]  

(7b)

From the Von-Mises theory, the diameter of the shaft can be calculated as follows:

\[ \frac{\sigma_{\text{eq}}}{F_s} = k \sqrt{\frac{2}{3} \sigma_b^2 + \frac{3}{4} \tau^2} \]  

(8)

Manipulating equation (8) the shaft diameter can be obtained with the help of the following equation:

\[ d = \sqrt[3]{\frac{3 kF_s \sqrt{M_b^2+0.75T^2}}{\pi \sigma_{\text{eq}}}} \]  

(9)

The shaft diameter can be optimized by applying a number of iteration until shaft diameter reaches minimum with ability to carry given magnitude of loading condition.

**Bearing Design:**

The radial loads acting on the bearing can be calculated using the shear diagram calculated before. The design load is the maximum load of the two reaction force that acts on the bearing. Bearings on the shaft will be designed based on the maximum reaction force on individual bearing. The theory was Found from *Bearing Design in Machinery: Engineering Tribology and Lubrication*. The methodology of designing a ball bearing is shown as follows:

From shaft diameter, the inner diameter of the bearing raceway can be calculated as equal to the shaft diameter. The outer race radius would then equal to the inner race radius plus the diameter of the rolling element i.e. the ball. The bearing can be designed as follows:

Considering the high speed of the shaft operation, the centrifugal forces acting on the outer raceway of the bearing must be considered for the shaft design.

The equivalent radius of the bearing in x-direction can be calculated as follows:

\[ \frac{1}{R_e} = \frac{1}{R_{e1}} + \frac{1}{R_{e2}} \]  

(10)
Similarly, the equivalent radius of the bearing in y-direction can be calculated as follows:

\[
\frac{1}{R_y} = \frac{1}{R_{1y}} - \frac{1}{R_{2y}}
\]  

(11)

The equivalent radius of the bearing can be calculated as follows:

\[
\frac{1}{R_{eq}} = \frac{1}{R_e} + \frac{1}{R_i}
\]  

(12)

The ratio of radius in y and x direction respectively, is an important factor in calculating the ellipsoidal parameters of the contact area. The \( \alpha \) of the given bearing can be calculated as follows:

\[
\alpha = \frac{R_y}{R_e}
\]  

(13)

The factor \( k \) can be calculated as follows:

\[
k \approx \alpha^2
\]  

(14)

Further,

\[
q_a = \frac{5}{2} - 1 = 0.57
\]  

(15)

\[
\hat{E} \approx 1 + \frac{q_a}{\alpha}
\]  

(16)

The equivalent Modulus of elasticity for same material of raceways and balls is shown as follows:

\[
E_{eq} = \frac{E}{1 - \nu^2}
\]  

(17)

The maximum load on a bearing can be calculated as follows:

\[
W_{max} = \frac{5 W_{bearing}}{n_r}
\]  

(18)

The ellipsoid parameters can be calculated as follows:

\[
a = \left(\frac{6\hat{E} W_{max} R_{eq}}{\pi k E_{eq}}\right)^{\frac{1}{4}}
\]  

(19)

\[
b = \left(\frac{6\hat{E} W_{max} k^2 R_{eq}}{\pi E_{eq}}\right)^{\frac{1}{4}} = k a
\]  

(20)

The bearing speed is considerable high. For a high speed application, centrifugal force of the balls acting on the outer race of the bearing must be considered to obtain correct maximum contact pressure acting on the outer race of the bearing. Centrifugal force acting on the outer ring can be calculated as follows:

\[
F_c = m\omega^2 (R_{in} + R_{ball})
\]  

(21)
Where,

\[ \omega_c = \frac{R_{in}}{R_{in}^2 + R_{out}^2} \omega ; \quad m = \rho V = \rho \left( \frac{\pi d^3}{6} \right) \quad (22) \]

The equivalent maximum loading on the contact area is shown as follows;

\[ W_{eq} = W_{max} + F_c \quad (23) \]

The Maximum pressure on the outer race of the bearing can be calculated as follows:

\[ P_{max} = \frac{3 W_{eq}}{2 \pi ab} \quad (24) \]

A number of combination and permutations of the ball diameter and number of balls can be applied to minimize the maximum contact pressure on the outer race of the bearing.

**Determining the Minimum film thickness:** Found from *Bearing Design in Machinery: Engineering Tribology and Lubrication*.

The rolling speed can be found as follows:

\[ U_r = \frac{R(R_c + d)}{R_c + d} \omega \quad (25) \]

Dimensionless rolling speed is:

\[ U_r = \frac{\nu_0}{E_{eq} R_c} U_r \quad (26) \]

The Dimensionless maximum load can be expressed as follows:

\[ \overline{W} = \frac{W_{max}}{E_{eq} R_c} \quad (27) \]

The minimum film thickness can be found as follows:

\[ \frac{h_{min}}{R_c} = 3.63 \overline{U_r}^{0.68} (\alpha E_{eq})^{0.49} (1 - e^{-0.68k}) \quad (28a) \]

\[ h_{min} = 3.63 R_x \overline{U_r}^{0.68} (\alpha E_{eq})^{0.49} (1 - e^{-0.68k}) \quad (28b) \]
**Gear Design:**

There are three major failure theories for gear failure. They are namely the bending stress failure which is similar to that of bending of a beam, the other failure theory is pitting failure and the third one is scalp removal failure theory (according to this theory, a layer of chunks of gear teeth get failed under stress). For the application of this project, the bending stress theory for gear failure is utilized, it takes into account the bending failure of gear tooth. The Theory was obtained from *Design of Machine Elements, 8th Edition*, Chapter 10 Spur Gears.

**Bending Strength:**

The bending strength of a gear can be found out using the following equation:

$$S_t = W_o K_o K_v K_s \frac{P_d K_m K_b}{F}$$

(29)

Where,

- $S_t$ = Bending strength
- $W_o$ = transmitted tangential load
- $K_o$ = Overload factor
- $K_v$ = Dynamic Factor
- $K_s$ = Size factor
- $P_d$ = Diametral Pitch
- $F$ = Net Face Width
- $K_m$ = Load Distribution Factor
- $K_b$ = Rim thickness factor
- $J$ = Geometry Factor

The gear can be designed as per the bending strength theory as shown below:

$$S_t \leq \frac{S_{at} Y_N}{S_f K_T K_R}$$

(30)

- $S_{at}$ = Allowable bending strength
- $Y_N$ = The cycle stress factor
- $S_f$ = Safety Factor for bending strength
- $K_T$ = The temperature Factor
- $K_R$ = the reliability factor

Note that all the above calculations for Gear bending strength should be performed in English Units. However, the calculations for shaft design and the bearing were done using SI units.
**Analysis and Calculations:**

**Shaft 1:**

Shaft 1 is the power input shaft. A power of 2000 KW at speed of 6000 RPM is supplied by external motor. Shaft 1 holds one gear and two bearings at each end respectively. This can be shown with the help of following figure.

![Figure: A diagram showing loading condition on shaft 1.](image)

**Gear Radial and Tangential force Calculation:**

The tangential force and the radial force acting on the gear tooth is given as: (Note: $\phi$ = Pressure angle of the gear typically $20^\circ$). Let the gear diameter be 300mm, Radius of the gear 1, $R=150\text{mm}$.

\[
\omega = \frac{2\pi N}{60} = \frac{2(6000\text{RPM})}{60} = 200\pi\text{rad/s} = 628.318\text{rad/s}
\]

\[
F_t = \frac{\dot{E}}{\omega R} = \frac{2000\text{KW}}{(628.318\text{rad/s})(0.150\text{m})} = 21,220\text{ N}
\]

\[
F_r = F_t\tan\phi = 7723.695\text{ N}
\]

The radial force acting on the shaft is given as:

\[
F_{tr} = \sqrt{F_t^2 + F_r^2} = 22,582.57\text{ N}
\]

The torque acting on the shaft as a result of the power transmission is given as follows:

\[
T = \frac{\dot{E}}{\omega} = 3183.09\text{ Nm}
\]
The shaft (Equivalent beam) diagram can be solved to obtain the maximum bending moment and shear force diagrams to find the reaction forces acting on the bearings at both the ends of the shaft.

![Bending-Moment Diagram](image.png)

Figure: A Bending-Moment diagram for shaft 1.

From the bending moment diagram, the maximum bending moment can be utilized to calculate the maximum bending stress acting on the shaft. The maximum bending moment is \( M = 5419.92 \) Nm. The maximum shear stress occurs on the outer surface of the shaft. The torque
equals \( T = 3183.1 \) Nm. Other required properties for shaft diameter calculation are shown as follows:

Properties taken from references:

\( \sigma_{yp} = 560 \, MPa \) for AISI 1050 Steel: Medium Carbon Shaft Steel).
\( F_s = 2 \) (User-defined)
\( k = 1.6 \) For Profile key way of Medium Carbon steel taken From *Machine Design* Book.

Using equation (9) the shaft diameter can be obtained with the help of the following equation:

\[
d = \sqrt{\frac{kF_s \sigma_{yp}}{\pi \sigma_{yp}}} \sqrt{\frac{\pi^2 + 0.75 T^2}{\pi^2 R^*}}
\]

\[
d = \sqrt{\frac{(1.6)(2) \sqrt{(5419.92 Nm)^2 + 0.75(3183.1 Nm)^2}}{\pi(580 \, MPa)}}
\]

\[
d = 65 \, mm
\]

The shaft diameter can be optimized by applying a number of iteration until shaft diameter reaches minimum with ability to carry given magnitude of loading condition. The optimized shaft diameter was found to be about 65mm rounded to

*Bearing design:*

Known Parameters:

Inner raceway diameter of the bearing \( 1 = D_{in} = 65 \, mm \)
Inner raceway radius of the bearing \( 1 = R_{in} = 32.5 \, mm \)
Let the ball diameter be \( D_{Ball} = 10 \, mm; R_{Ball} = 5 \, mm \)

Then the number of roller balls in the bearing can be computed as follows;

\[
n_r = \frac{\pi (R_{in} + R_{Ball})}{1.5 R_{Ball}} = \frac{\pi (32.5 \, mm + 5 \, mm)}{1.5 (5 \, mm)} = 15.7
\]

We round the number of balls down. Thus number of balls in the bearing are about 15.

The equivalent radius of the bearing in x-direction can be calculated as follows:

\[
\frac{1}{R_x} = \frac{1}{R_{1x}} + \frac{1}{R_{2x}} = \frac{1}{32.5 \, mm} + \frac{1}{5 \, mm}
\]

\[
R_x = 4.3333 \, mm
\]
Similarly, the equivalent radius of the bearing in y-direction can be calculated as follows:

\[
\frac{1}{R_y} = \frac{1}{R_{1y}} - \frac{1}{R_{2y}} = \frac{1}{5\text{mm}} - \frac{1}{5.1\text{mm}}
\]

\[R_y = 255 \text{ mm}\]

The equivalent radius of the bearing can be calculated as follows:

\[
\frac{1}{R_{eq}} = \frac{1}{R_x} + \frac{1}{R_y} = \frac{1}{4.33\text{mm}} + \frac{1}{255\text{mm}}
\]

\[R_{eq} = 4.27 \text{ mm}\]

The ratio of radius in y and x direction respectively, is an important factor in calculating the ellipsoidal parameters of the contact area. The α of the given bearing can be calculated as follows:

\[\alpha = \frac{R_y}{R_x} = 58.85\]

The factor k can be calculated as follows:

\[k \approx \alpha^2 = 13.34\]

Further,

\[q_a = \frac{5}{2} - 1 = 0.57\]

\[\hat{E} \approx 1 + \frac{q_a}{\alpha} = 1.043\]

The equivalent Modulus of elasticity for same material of raceways and balls is shown as follows:

\[E_{eq} = \frac{E}{1-\theta^2} = \frac{2\times10^{11}}{1-(0.3)^2} = 2.2\times10^{11} \text{ Pa}\]

The maximum load on a bearing can be calculated as follows:

\[W_{max} = \frac{5W_{bearing}}{n_r} = \frac{5(13549\text{N})}{15} = 4516.33 \text{ N}\]

The ellipsoid parameters can be calculated as follows:

\[a = \left(\frac{6\hat{E} W_{max} R_{eq}}{nk E_{eq}}\right)^{\frac{1}{3}} = 0.00002 \text{ m} = 200 \mu \text{m}\]

\[b = \left(\frac{6\hat{E} W_{max} k^2 R_{eq}}{n E_{eq}}\right)^{\frac{1}{3}} = ka = 0.0031 = 3.1 \text{ mm}\]

The bearing speed is considerable high. For a high speed application, centrifugal force of the balls acting on the outer race of the bearing must be considered to obtain correct maximum
contact pressure acting on the outer race of the bearing. Centrifugal force acting on the outer ring can be calculated as follows:

Where,

\[ \omega_c = \frac{R_{in}}{R_{in} + R_{out}} \omega = 272.27 \text{ rad/s} \]

\[ m = pV = \rho \left( \frac{\pi d_{ball}^3}{6} \right) = 0.5105 \text{ g} = 0.0005105 \text{ kg} \]

\[ F_c = m \omega_c^2 (R_{in} + R_{ball}) = 5.676 \text{ N} \]

The equivalent maximum loading on the contact area is shown as follows;

\[ W_{eq} = W_{max} + F_c = 4522 \text{ N} \]

The Maximum pressure on the outer race of the bearing can be calculated as follows:

\[ P_{max} = \frac{3}{2} \frac{W_{eq}}{\pi d_{ball}^2} = 2.94 \text{ GPa} \]

A number of combination and permutations of the ball diameter and number of balls can be applied to minimize the maximum contact pressure on the outer race of the bearing.

**Determining the Minimum film thickness:**

The rolling speed can be found as follows:

\[ U_r = \frac{R_s (R_s + d)}{R_s + d} \omega \left( \frac{0.065m (0.065m + 0.01)}{2 (0.065m + 0.01)} \right) (628) = 21.88 \text{ m/s} \]

\[ \overline{U_r} = \frac{\mu_0}{E_{eq} R_s} U_r = 2.07 \times 10^{-10} \]

The Dimensionless maximum load can be expressed as follows:

\[ \overline{W} = \frac{W_{max}}{E_{eq} R_s^2} = \frac{(4522 \text{ N})}{(2.2 \times 10^{-10})(0.0048m)^2} = 8.92 \times 10^{-4} \]

The minimum film thickness can be found as follows:

\[ h_{min} = 3.63 R_s \overline{U_r}^{0.68} (a E_{eq})^{0.49} \left( 1 - e^{-0.68k} \right) \]

\[ h_{min} = 3.63 (0.0048m) \left( \frac{2.07 \times 10^{-10}}{0.000892} \right)^{0.49} \left( 1 - e^{-0.68(13.34)} \right) \]

\[ h_{min} = 0.494 \mu m \]

The minimum film thickness for this bearing is about 0.49 \( \mu m \)
**Shaft 2:**

Shaft 2 is the power input shaft. A power of 2000 KW at speed of 2400 RPM is supplied by external motor. Shaft 1 holds one gear and two bearings at each end respectively. This can be shown with the help of following figure.

![Load Diagram](image)

**Figure:** A diagram showing loading condition on shaft 2.

**Gear Radial and Tangential force Calculation:**

The tangential force and the radial force acting on the gear tooth is given as: (Note: \( \varphi = \) Pressure angle of the gear typically 20\(^\circ\)). Let the gear diameter be 300mm, Radius of the gear 1, \( R = 150 \)mm.

\[
\omega = \frac{2\pi \times N}{60} = \frac{2\pi \times 6000 \text{ RPM}}{2.5 \times 60} = 80\pi \text{ rad} = 251.327 \text{ rad}
\]

\[
F_t = \frac{\dot{E}}{\omega R} = \frac{2000 \text{ KW}}{(628.31 \text{ rad})(0.150 \text{ m})} = 53,051 \text{ N}
\]

\[
F_r = F_t \tan \varphi = 19,310 \text{ N}
\]

The radial force acting on the shaft is given as:

\[
F_{tr} = \sqrt{F_t^2 + F_r^2} = 56878 \text{ N}
\]

The torque acting on the shaft as a result of the power transmission is given as follows:

\[
T = \frac{\dot{E}}{\omega} = 7957.725 \text{ Nm}
\]

The shaft (Equivalent beam) diagram can be solved to obtain the maximum bending moment and shear force diagrams to finds the reaction forces acting on the bearings at both the ends of the shaft.
From the bending moment diagram, the maximum bending moment can be utilized to calculate the maximum bending stress acting on the shaft. The maximum bending moment is $M = 5419.92 \text{ Nm}$. The maximum shear stress occurs on the outer surface of the shaft. The torque equals $T = 3183.1 \text{ Nm}$. Other required properties for shaft diameter calculation are shown as follows:

Properties taken from references:

$\sigma_{yp} = 560 \text{ MPa}$ for AISI 1050 Steel: Medium Carbon Shaft Steel).
\( F_s = 2 \) (User-defined)
\( k = 1.6 \) For Profile key way of Medium Carbon steel taken From Machine Design Book.

Using equation (9) the shaft diameter can be obtained with the help of the following equation:

\[
\begin{align*}
    d &= \sqrt[3]{\frac{3kF_s \sqrt{M_p^2 + 0.75T^2}}{\pi \sigma_y}} \\
    d &= \sqrt[3]{\frac{(1.6)(2)(26421N/m)^2 + 0.75(7957 N/m)^2}{\pi(580MPa)}} \\
    d &= 105 \text{ mm}
\end{align*}
\]

The shaft diameter can be optimized by applying a number of iteration until shaft diameter reaches minimum with ability to carry given magnitude of loading condition. The optimized shaft diameter was found to be about 105mm.

**Bearing design:**

Known Parameters:

Inner raceway diameter of the bearing 1 = \( D_{in} = 105 \) mm
Inner raceway radius of the bearing 1 = \( R_{in} = 52.5 \) mm
Let the ball diameter be \( D_{ball} = 10 \) mm; \( R_{ball} = 7.5 \) mm

Then the number of roller balls in the bearing can be computed as follows;

\[
\begin{align*}
    n_r &= \pi (R_{in} + R_{ball}) \frac{1}{1.5R_{ball}} \\
    n_r &= \pi (52.5mm + 7.5mm) \frac{1}{1.5(5mm)} \\
    n_r &= 14.5
\end{align*}
\]

We round the number of balls down. Thus number of balls in the bearing are about 14.

The equivalent radius of the bearing in x-direction can be calculated as follows:

\[
\begin{align*}
    \frac{1}{R_x} &= \frac{1}{R_{1x}} + \frac{1}{R_{2x}} \\
    \frac{1}{R_x} &= \frac{1}{52.5mm} + \frac{1}{7.5mm} \\
    R_x &= 6.5625 \text{ mm}
\end{align*}
\]

Similarly, the equivalent radius of the bearing in y-direction can be calculated as follows:

\[
\begin{align*}
    \frac{1}{R_y} &= \frac{1}{R_{1y}} - \frac{1}{R_{2y}} \\
    \frac{1}{R_y} &= \frac{1}{7.5mm} - \frac{1}{7.75mm} \\
    R_y &= 230 \text{ mm}
\end{align*}
\]
The equivalent radius of the bearing can be calculated as follows:

\[
\frac{1}{R_{eq}} = \frac{1}{R_y} + \frac{1}{R_x} = \frac{1}{6.5625\,mm} + \frac{1}{230\,mm}
\]

\[
R_{eq} = 6.38\,mm
\]

The ratio of radius in y and x direction respectively, is an important factor in calculating the ellipsoidal parameters of the contact area. The \( \alpha \) of the given bearing can be calculated as follows:

\[
\alpha = \frac{R_y}{R_x} = 35.047
\]

The factor \( k \) can be calculated as follows:

\[
k \approx \alpha^{\frac{2}{3}} = 9.61
\]

Further,

\[
\hat{E} \approx 1 + \frac{q_a}{\alpha} = 1.059
\]

The equivalent Modulus of elasticity for same material of raceways and balls is shown as follows:

\[
E_{eq} = \frac{E}{1-\nu^2} = \frac{2 \times 10^{11}}{1-(0.3)^2} = 2.2 \times 10^{11}\, Pa
\]

The maximum load on a bearing can be calculated as follows. Note that the design radial force for bearing is 44035N. It can be found from the shear force diagram.

\[
W_{max} = \frac{5W_{bearing}}{n_r} = \frac{5(44035\,N)}{14} = 15726.78\,N
\]

The ellipsoid parameters can be calculated as follows:

\[
a = \left( \frac{6\hat{E}W_{max}R_y}{\pi kE_{eq}} \right)^{\frac{1}{3}} = 9.797\,\mu m
\]

\[
b = \left( \frac{6\hat{E}W_{max}k^2R_x}{\pi E_{eq}} \right)^{\frac{1}{3}} = ka = 94.15\,\mu m
\]

The bearing speed is considerable high. For a high speed application, centrifugal force of the balls acting on the outer race of the bearing must be considered to obtain correct maximum contact pressure acting on the outer race of the bearing. Centrifugal force acting on the outer ring can be calculated as follows:

Where,
\[ \omega_c = \frac{R_{in}}{R_{in} + R_{out}} \omega = 117.3 \text{ rad/s} \]

\[ m = \rho V = \rho \left( \frac{\pi d_{ball}}{6} \right) = 1.722 \text{ g} = 0.0017 \text{ kg} \]

\[ F_c = m \omega_c^2 (R_{in} + R_{ball}) = 2.84 \text{ N} \]

The equivalent maximum loading on the contact area is shown as follows;

\[ W_{eq} = W_{max} + F_c = 15730 \text{ N} \]

The Maximum pressure on the outer race of the bearing can be calculated as follows:

\[ P_{max} = \frac{3 W_{eq}}{2 \pi ab} = 3.65 \text{ GPa} \]

A number of combination and permutations of the ball diameter and number of balls can be applied to minimize the maximum contact pressure on the outer race of the bearing.

**Determining the Minimum film thickness:**

The rolling speed can be found as follows:

\[ U_r = \frac{R_{in} \omega}{k_{oct} + d} \approx \frac{(0.065 m)(0.065 m + 0.01)}{2(0.065 m)^2} (628) = 14.074 \text{ m/s} \]

\[ \overline{U}_r = \frac{b_0}{E_{eq} k_{oct}} U_r = 9.804 \times 10^{-11} \]

The Dimensionless maximum load can be expressed as follows:

\[ \overline{W} = \frac{W_{max}}{E_{eq} k_{oct}} = \frac{(15730 \text{ N})}{(2.2 \times 10^{15})(0.0048 \text{ m})} = 0.001679 \]

The minimum film thickness can be found as follows:

\[ h_{min} = 3.63 R_{in} \left( \frac{U_r^{0.68} (a E_{eq})^{0.49}}{\overline{W}^{0.0073}} \right) (1 - e^{-0.68k}) \]

\[ h_{min} = 3.63(0.006525 m) \left( \frac{(9.804 \times 10^{-11})^{0.68}}{(35.047)(5.06 \times 10^3)^{0.49}} \right) (0.001679^{0.0073}) \left(1 - e^{-0.68(0.61)} \right) \]

\[ h_{min} = 0.369 \mu m \]

The minimum film thickness for this bearing is about 0.369 \mu m
**Shaft 3:**

Shaft 3 is the power input shaft. A power of 2000 kW at speed of 960 RPM is supplied by external motor. Shaft 1 holds one gear and two bearings at each end respectively. This can be shown with the help of following figure.

![Shaft 3 Diagram](image)

Figure: A diagram showing loading condition on shaft 3.

**Gear Radial and Tangential force Calculation:**

The tangential force and the radial force acting on the gear tooth is given as: (Note: $\phi = \text{Pressure angle of the gear typically 20}^\circ$). Let the gear diameter be 300mm, Radius of the gear 1, $R=150$mm.

$$\omega = \frac{2\pi N}{60} = \frac{2\pi (6000 \text{RPM})}{6.25 \times 60} = 32\pi \text{rad/s} = 100.53 \text{ rad/s}$$

$$F_t = \frac{E}{\omega R} = \frac{2000 \text{KW}}{(628.318 \text{rad/s})(0.150 \text{ m})} = 53,051 \text{ N}$$

$$F_r = F_t \tan \phi = 19,310 \text{ N}$$

The radial force acting on the shaft is given as:

$$F_{tr} = \sqrt{F_t^2 + F_r^2} = 56878 \text{ N}$$

The torque acting on the shaft as a result of the power transmission is given as follows:

$$T = \frac{E}{\omega} = 19894 \text{ Nm}$$

The shaft (Equivalent beam) diagram can be solved to obtain the maximum bending moment and shear force diagrams to find the reaction forces acting on the bearings at both the ends of the shaft.
From the bending moment diagram, the maximum bending moment can be utilized to calculate the maximum bending stress acting on the shaft. The maximum bending moment is $M = 5419.92$ Nm. The maximum shear stress occurs on the outer surface of the shaft. The torque equals $T = 3183.1$ Nm. Other required properties for shaft diameter calculation are shown as follows:

Properties taken from references:

$\sigma_{yp} = 560 \, MPa$ for AISI 1050 Steel: Medium Carbon Shaft Steel).

$F_s = 2$ (User-defined)

$k = 1.6$  For Profile key way of Medium Carbon steel taken From Machine Design Book.
Using equation (9) the shaft diameter can be obtained with the help of the following equation:

\[ d = \sqrt{\frac{kF_s\sqrt{M_p^2 + 0.75T^2}}{\pi\sigma_{yp}}}, \]

\[ d = \sqrt{\frac{(1.6)(2)\sqrt{(13549\text{Nm})^2 + 0.75(19894\text{Nm})^2}}{\pi(580\text{MPa})}}, \]

\[ d = 105 \text{ mm} \]

The shaft diameter can be optimized by applying a number of iteration until shaft diameter reaches minimum with ability to carry given magnitude of loading condition. The optimized shaft diameter was found to be about 105mm.
Results and Summary

Bearing design:

Known Parameters:

Inner raceway diameter of the bearing 1 = \( D_{in} = 105 \) mm  
Inner raceway radius of the bearing 1 = \( R_{in} = 52.5 \) mm  
Let the ball diameter be \( D_{Ball} = 10 \) mm; \( R_{Ball} = 7.5 \) mm

Then the number of roller balls in the bearing can be computed as follows;

\[
R_f = \frac{\pi (R_{in} + R_{ball})}{1.5 R_{ball}} = \frac{\pi (52.5 mm + 7.5 mm)}{1.5 (5mm)} = 14.5
\]

We round the number of balls down. Thus number of balls in the bearing are about 14.

The equivalent radius of the bearing in x-direction can be calculated as follows:

\[
\frac{1}{R_x} = \frac{1}{R_{1x}} + \frac{1}{R_{2x}} = \frac{1}{52.5 mm} + \frac{1}{7.5 mm} \\
R_x = 6.5625 \text{ mm}
\]

Similarly, the equivalent radius of the bearing in y-direction can be calculated as follows:

\[
\frac{1}{R_y} = \frac{1}{R_{1y}} - \frac{1}{R_{2y}} = \frac{1}{7.5 mm} - \frac{1}{7.75 mm} \\
R_y = 230 \text{ mm}
\]

The equivalent radius of the bearing can be calculated as follows:

\[
\frac{1}{R_{eq}} = \frac{1}{R_x} + \frac{1}{R_y} = \frac{1}{6.5625 mm} + \frac{1}{230 mm} \\
R_{eq} = 6.38 \text{ mm}
\]

The ratio of radius in y and x direction respectively, is an important factor in calculating the ellipsoidal parameters of the contact area. The \( \alpha \) of the given bearing can be calculated as follows:

\[
\alpha = \frac{R_y}{R_x} = 35.047
\]

The factor \( k \) can be calculated as follows:

\[
k \approx \alpha^{\frac{3}{2}} = 9.61
\]

Further,
\[ q_a = \frac{2}{\pi} - 1 = 0.57 \]
\[ \hat{E} \approx 1 + \frac{q_a}{\alpha} = 1.059 \]

The equivalent Modulus of elasticity for same material of raceways and balls is shown as follows:
\[ E_{eq} = \frac{E}{1-q_a^2} = \frac{2 \times 10^{11}}{1-(0.5)^2} = 2.2 \times 10^{11} \text{ Pa} \]

The maximum load on a bearing can be calculated as follows. Note that the design radial force for bearing is 33873N. It can be found from the shear force diagram.
\[ W_{\text{max}} = \frac{5W_{\text{bearing}}}{n_r} = \frac{5(33873 \text{ N})}{14} = 12097.5 \text{ N} \]

The ellipsoid parameters can be calculated as follows:
\[ a = \left( \frac{6\hat{E} W_{\text{max}} R_{eq}}{\pi k E_{eq}} \right)^{\frac{1}{2}} = 416.677 \mu m \]
\[ b = \left( \frac{6\hat{E} W_{\text{max}}^2 R_{eq}}{\pi E_{eq}} \right)^{\frac{1}{2}} = ka = 4 \text{ mm} \]

The bearing speed is considerable high. For a high speed application, centrifugal force of the balls acting on the outer race of the bearing must be considered to obtain correct maximum contact pressure acting on the outer race of the bearing. Centrifugal force acting on the outer ring can be calculated as follows:

Where,
\[ \omega_c = \frac{R_{in}}{R_{in} + R_{out}} \omega = 46.91 \text{ rad/s} \]
\[ m = \rho V = \rho \left( \frac{\pi d_{ball}^3}{6} \right) = 1.722 \text{ g} = 0.0017 \text{ kg} \]
\[ F_c = m \omega_c^2 (R_{in} + R_{ball}) = 2.13 \text{ N} \]

The equivalent maximum loading on the contact area is shown as follows;
\[ W_{eq} = W_{\text{max}} + F_c = 12100 \text{ N} \]

The Maximum pressure on the outer race of the bearing can be calculated as follows:
\[ P_{\text{max}} = \frac{3W_{eq}}{2\pi ab} = 3.79 \text{ GPa} \]

A number of combination and permutations of the ball diameter and number of balls can be applied to minimize the maximum contact pressure on the outer race of the bearing.
**Determining the Minimum film thickness:**

The rolling speed can be found as follows:

\[
U_r = \frac{R_d(R_e + d)}{R_e + d} \omega \left(\frac{0.065m(0.065m+0.01)}{2(0.065m)+0.01}\right)(100.5) = 3.5175m/s
\]

\[
\bar{U}_r = \frac{\mu_E}{EeqK_L}U_r = 2.55 \times 10^{-11}
\]

The Dimensionless maximum load can be expressed as follows:

\[
\bar{W} = \frac{W_{max}}{EeqK_L} = \frac{(12100N)}{(2.2 \times 10^{11})(0.00625m^2)} = 0.001408
\]

The minimum film thickness can be found as follows:

\[
h_{min} = 3.63R_x \bar{U}_r^{0.68} (\alpha E_{eq})^{0.49} \left(1 - e^{-0.68k}\right)
\]

\[
h_{min} = 3.63(0.00625m)\left(2.55 \times 10^{-11}\right)^{0.68}(2.12(2.2 \times 10^{3}))^{0.49} \left(1 - e^{-0.68(9.61)}\right)
\]

\[
h_{min} = 0.154 \mu m
\]

The minimum film thickness for this bearing is about 0.154 \mu m

---

**Gear Design:**

**Design for Gear 1:**

**Bending Strength:**

The bending strength of a gear can be found out using the following equation:

\[
S_t = W_tK_oK_vK_sP_dK_wK_bF
\]

Where,

\[S_t = \text{Bending strength}\]
\[W_t = \text{transmitted tangential load} = 22,582 \text{ N} = 5076.6 \text{lbf}\]
\[K_o = \text{Overload factor} = 1\]
\[K_v = \text{Dynamic Factor} = 1.25 \text{ (@ 515.3 ft/s from Figure 10.29 Design of Machine elements)}\]
\[K_s = \text{Size factor} = 1\]
\[P_d = \text{Diametral Pitch}\]
\[F = \text{Net Face Width} = 0.2 \text{ m} = 7.87 \text{ in.}\]
\[K_w = \text{Load Distribution Factor} = 2.025\]
\[K_b = \text{Rim thickness factor} = 1\]
\[J = \text{Geometry Factor} = 0.29 \text{ (From table 10.4)}\]
Thus,

\[ S_t = \frac{(5076.6 \text{ lbf})(1)(1.25)(1)\left(\frac{2.743 \text{ in.}}{2.025 \text{ in.}}\right)
\left(\frac{0.0787 \text{ in.}}{0.29}\right)}{\left(\frac{7.87 \text{ in.}}{0.7}\right)} = 15,442 \text{ psi} \]

The gear can be designed as per the bending strength theory as shown below:

\[ S_t \leq S_y Y_N \]

The material for gear is ASTM AS36 (Ductile Nodular Iron)

- \( S_{at} \): Allowable bending strength =25,000 Psi
- \( Y_N \): The cycle stress factor = 1.25
- \( S_F \): Safety Factor for bending strength =2
- \( K_T \): The temperature Factor = 1
- \( K_R \): the reliability factor = 1

\[ S_{at} Y_N = \frac{(25000 \text{ psi})(1.25)}{2(1.1)} = 15,625 \text{ psi} \]

The above condition is true. Thus the gear design is correct. Hence the proposed gear is selected for the given gearbox design. Similarly, the design for gear 2, 3 and 4 was performed and the results are displayed in Summary Table.

**Summary Table:**

<table>
<thead>
<tr>
<th>Gear</th>
<th>Material</th>
<th>Bending strength</th>
<th>Allowable strength</th>
<th>Face Width (m)</th>
<th>Pitch Diameter (Dp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear 1</td>
<td>ASTM AS36</td>
<td>15,442 Psi</td>
<td>15625 psi</td>
<td>0.2 m</td>
<td>0.3 m</td>
</tr>
<tr>
<td>Gear 2</td>
<td>ASTM AS36</td>
<td>155000 Psi</td>
<td>15625 Psi</td>
<td>0.2 m</td>
<td>0.750 m</td>
</tr>
<tr>
<td>Gear 3</td>
<td>ASTM AS36</td>
<td>17583 Psi</td>
<td>25000 Psi</td>
<td>0.2 m</td>
<td>0.3 m</td>
</tr>
<tr>
<td>Gear 4</td>
<td>ASTM AS36</td>
<td>18000 Psi</td>
<td>25000 Psi</td>
<td>0.2 m</td>
<td>0.750 m</td>
</tr>
</tbody>
</table>

The shaft Design parameters are defined as follows:

<table>
<thead>
<tr>
<th>Shaft</th>
<th>Material</th>
<th>Smaller diameter</th>
<th>Larger diameter</th>
<th>Length of Shaft</th>
<th>Extended length</th>
<th>Keyway Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft 1</td>
<td>AISI 1050</td>
<td>65 mm</td>
<td>85 mm</td>
<td>1 m</td>
<td>0.25 m</td>
<td>Profile</td>
</tr>
<tr>
<td>Shaft 2</td>
<td>AISI 1050</td>
<td>105 mm</td>
<td>120 mm</td>
<td>2 m</td>
<td>0</td>
<td>Profile</td>
</tr>
<tr>
<td>Shaft 3</td>
<td>AISI 1050</td>
<td>105 mm</td>
<td>120 mm</td>
<td>1 m</td>
<td>0.25 m</td>
<td>Profile</td>
</tr>
</tbody>
</table>
Figure: A summary table of Shaft Design

Bearing Design:

<table>
<thead>
<tr>
<th>Bearing</th>
<th>Material</th>
<th>Inner Diameter</th>
<th>Approximate Number of Balls</th>
<th>Maximum Load</th>
<th>Maximum Contact Pressure</th>
<th>Minimum Film Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing 1</td>
<td>Steel</td>
<td>65 mm</td>
<td>15</td>
<td>13,549 N</td>
<td>2.94 GPa</td>
<td>0.49 µm</td>
</tr>
<tr>
<td>Bearing 2</td>
<td>Steel</td>
<td>105 mm</td>
<td>14</td>
<td>44,025 N</td>
<td>3.65 GPa</td>
<td>0.37 µm</td>
</tr>
<tr>
<td>Bearing 3</td>
<td>Steel</td>
<td>105 mm</td>
<td>14</td>
<td>33,879 N</td>
<td>3.79 GPa</td>
<td>0.15 µm</td>
</tr>
</tbody>
</table>

Note: The above summarized values are proposed from the theory. However, actual design may have values varying slightly from proposed design. This is because the Bearing, Shaft and Gear catalog may have different dimensions than the proposed one.


**Results and Discussion:**

The transmission assembly that was created was based off of equations of max stress that were used to calculate geometries and geometric limitations that have been provided. The shaft’s geometric features were designed based on shear stress and geometric ratios of the gearbox. The gears’ geometric features like thickness and number of teeth were designed based on max stress and the overall gear ratio of the transmission as well as the shaft diameters. The gears were mounted to the shafts using woodruff keys. The bearings were chosen by the results of the shaft diameters, shaft rotational speeds, and max loads on the shaft; their models were designed based off the manufacturer’s description of the bearings that were selected for use in the transmission. The bearings are mounted to brackets towards the center of the transmission or plates towards the outside of the transmission as well as their designated shafts.

The same oil is used to lubricate both the gears and bearings in this transmission. A case for the transmission was designed with the intent to cool the oil that is used to lubricate the gears and bearings because friction alone is enough to substantially heat up the oil so that its viscosity might change. The casing is surrounded with cooling ribs for convectional heat transfer. The oil exits at the bottom right front corner of the transmission and re-enters at the top left back corner of the transmission so that the inlet is at the maximum distance from the outlet allowing the cool oil to travel completely through the transmission and heat up thoroughly before it is sent out to be cooled again.

In the oil cooling line the oil first enters through an oil filter necessary to absorb any particles in the oil as a result of both contamination and wear to the gears or bearings. The filtration of these particles is necessary to prevent them from damaging the sensitive tolerances needed for clean operation of the bearings and gears. Following the oil filter is a pump to push the oil back into the transmission and overcome gravity. Putting the filter before the pump also prevents contamination of the pump which may result in efficiency losses. Since shafts 1 and 3 are required to protrude outside of the transmission casing, additional plates with mounts for labyrinth seals have been designed to prevent oil from leaking out of the system.
Bearing Selection

When choosing bearings for the gearbox described above, it is very critical to take all the bearing rating factors into consideration. This includes the dynamic and static load rating, the limiting speed of the bearing (and shaft), and the bore diameter. In the design considered for this specific gearbox, the shaft diameters calculated were done so with a large safety factor. This means that when it came to selection of bearings there was some room to change the design. This came in very useful when looking through catalogs of radial ball bearings. All of the bore diameters are standardized. It is possible to create a bearing with a specific bore diameter that is not shown in the catalog. However, making a custom bearing could be extremely expensive. This is the reason that the shaft diameter were changed to fit the bearing rather than the changing the bearing diameter to fit the shaft.

The bearings chosen are as follows:

**Bearing 1**

Peer Bearing 6813  (radial ball bearing)
inner diameter: 65 mm 
outer diameter: 85 mm 
width: 10 mm 
Load Ratings: Dynamic- 12,000 N 
Static - 12,100 N
Limiting Speed: 8,700 RPM

**Bearing 2**

Peer Bearing 6826  (radial ball bearing)
inner diameter: 130 mm 
outer diameter: 165 mm
width: 18 mm
Load Ratings: Dynamic- 37,000 N
    Static - 44,000 N
Limiting Speed: 4,300 RPM

Bearing 3
Peer Bearing 6926 (radial ball bearing)
inner diameter: 130 mm
outer diameter: 180 mm
width: 24 mm
Load Ratings: Dynamic- 65,000 N
    Static - 67,500 N
Limiting Speed: 4,100 RPM

These bearing are strong enough to support the loads on them as well as the speeds they are operating at. (The relevant bearing catalog pages are shown in the appendix).

Shaft Selection

The shafts were created according to the calculations made and then modified according to the bore diameter of the bearings chosen. One of the most important part of the shaft, besides the lip made for the bearing to sit on, is the keyway. These keyways allow for transmission of the torque from the gear to the shaft and vice versa. The keyway dimensions are specified below.

Square woodruff key for shaft 1: width = 18mm
    depth = 4.4 mm
    length = 50mm
Square woodruff key for shaft 2: width= 32mm

depth= 7.4mm

length= 50mm

Square woodruff key for shaft 2: width= 32mm

depth= 7.4mm

length= 50mm

The shafts also have chamfers on the ends in order to facilitate the attachment of the bearings. This is a small but very useful design feature.
**BILL OF MATERIALS**

<table>
<thead>
<tr>
<th>PART NUMBER</th>
<th>DESCRIPTION</th>
<th>QTY</th>
<th>MATERIAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Shaft 1 with Keyway</td>
<td>1</td>
<td>AISI 1050 Steel</td>
</tr>
<tr>
<td>2</td>
<td>Key for Shaft 1</td>
<td>1</td>
<td>AISI 1050 Steel</td>
</tr>
<tr>
<td>3</td>
<td>&quot;Gear 1&quot; Metric - Spur gear 6M 50T 20PA 200FW ---S50N75H50L75R1</td>
<td>1</td>
<td>ASTM AS36 Steel</td>
</tr>
<tr>
<td>4</td>
<td>Shaft 3 with Keyway</td>
<td>1</td>
<td>AISI 1050 Steel</td>
</tr>
<tr>
<td>5</td>
<td>&quot;Gear 4&quot; Metric - Spur gear 6M 125T 20PA 200FW ---S125N75H50L120R1</td>
<td>1</td>
<td>ASTM AS36 Steel</td>
</tr>
<tr>
<td>6</td>
<td>Key for Shafts 2 and 3</td>
<td>3</td>
<td>AISI 1050 Steel</td>
</tr>
<tr>
<td>7</td>
<td>&quot;Gear 3&quot; Metric - Spur gear 6M 50T 20PA 200FW ---S50N75H50L120R1</td>
<td>1</td>
<td>ASTM AS36 Steel</td>
</tr>
<tr>
<td>8</td>
<td>&quot;Gear 4&quot; Metric - Spur gear 6M 125T 20PA 200FW ---S125N75H50L115R1</td>
<td>1</td>
<td>ASTM AS36 Steel</td>
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<tr>
<td>9</td>
<td>Shaft 2 with Keyway</td>
<td>1</td>
<td>AISI 1050 Steel</td>
</tr>
<tr>
<td>10</td>
<td>&quot;Shaft 1 Bearing&quot; Peer Bearing 6813  Radial Ball Bearing</td>
<td>2</td>
<td>Steel</td>
</tr>
<tr>
<td>11</td>
<td>&quot;Shaft 2 Bearing&quot; Peer Bearing 6826  Radial Ball Bearing</td>
<td>2</td>
<td>Steel</td>
</tr>
<tr>
<td>12</td>
<td>&quot;Shaft 3 bearing&quot; Peer Bearing 6926  Radial Ball Bearing</td>
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<td>Steel</td>
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<td>15</td>
<td>Shaft 2 Support</td>
<td>1</td>
<td>Steel</td>
</tr>
<tr>
<td>16</td>
<td>Shaft 2 and 3 Support and Cover</td>
<td>1</td>
<td>Steel</td>
</tr>
<tr>
<td></td>
<td>Description</td>
<td>Quantity</td>
<td>Material</td>
</tr>
<tr>
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<td>-------------------------------------------------------</td>
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<tr>
<td>17</td>
<td>Output Cover</td>
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<td>Steel</td>
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<tr>
<td>18</td>
<td>Labyrinth Seal Plate Output Side</td>
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<td>Steel</td>
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<tr>
<td>19</td>
<td>Labyrinth Seal Output Side</td>
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<td>Rubber</td>
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<td>20</td>
<td>Input Cover</td>
<td>1</td>
<td>Steel</td>
</tr>
<tr>
<td>21</td>
<td>Labyrinth Seal Input Side</td>
<td>1</td>
<td>Rubber</td>
</tr>
<tr>
<td>22</td>
<td>Labyrinth Seal Plate Input Side</td>
<td>1</td>
<td>Steel</td>
</tr>
<tr>
<td>23</td>
<td>B18.2.3.6M - Heavy hex bolt M20 x 2.5 x 110 --46N</td>
<td>12</td>
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<tr>
<td>24</td>
<td>Filter Holder Inlet</td>
<td>1</td>
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<tr>
<td>25</td>
<td>Oil Filter</td>
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<tr>
<td>26</td>
<td>Filter Holder Outlet</td>
<td>1</td>
<td>Steel</td>
</tr>
<tr>
<td>27</td>
<td>Pipe from Oil Filter to Pump</td>
<td>1</td>
<td>Steel</td>
</tr>
<tr>
<td>28</td>
<td>Pump</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>Pipe from Pump to Transmission Oil Supply Location</td>
<td>1</td>
<td>Steel</td>
</tr>
</tbody>
</table>
ASSEMBLY
TRANSMISSION

SECTION A-A

PEER BEARING 6813
Conclusion:

Given the new rotational velocity of the input shafts we were able to successfully select new bearings out of a catalog that could withstand the new loads and speeds of the shafts in the transmission. The shafts were designed using the Von Mises theory of failure and the loads imparted onto the bearings were calculated using the maximum bending moment and shear force. The transmission’s geometric features had to be modified to accommodate the new characteristics of the transmission. This also included selecting new shafts with different diameters as well as new gears to accommodate the new shaft sizes. A case and cooling system for the transmission has also been designed to provide the necessary lubrication that the bearings and gears need to operate smoothly and at the correct temperatures.
References:


Automobile Gearbox Image obtained from:


Applications of Gearbox found from:


Catalogs Found from:

For Spur Gears:


For Custom Spur Gears:

http://www.arrowgear.com/products/custom_gears.html

For Bearing:

### Nomenclature:

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<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tr>
<td>$\alpha$</td>
<td>The ratio of radius in y and x direction respectively</td>
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<tr>
<td>$\eta$</td>
<td>Efficiency of Power transmission here 100%</td>
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<tr>
<td>$\mu$</td>
<td>The viscosity of the fluid (oil)</td>
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<td>$\phi$</td>
<td>Pressure angle of the gear</td>
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<td>$\sigma$</td>
<td>Bending stress</td>
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<td>$\tau$</td>
<td>Shear stress</td>
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<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
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<tr>
<td>$\omega$</td>
<td>Angular speed</td>
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<tr>
<td>a</td>
<td>Shorter Ellipsoidal radii</td>
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<tr>
<td>b</td>
<td>larger Ellipsoidal radii</td>
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<tr>
<td>d</td>
<td>Diameter of the shaft</td>
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<td>F</td>
<td>Radial Bearing force acting on the shaft</td>
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<td>$F_s$</td>
<td>Factor of Safety</td>
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<td>h</td>
<td>minimum film thickness</td>
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<td>J</td>
<td>Geometry factor in gear</td>
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<tr>
<td>k</td>
<td>Various factors used in Gear bending calculations</td>
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<td>L</td>
<td>Length of the gear/ Bearing/ Shaft</td>
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<td>M</td>
<td>Bending moment of the shaft</td>
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<td>N</td>
<td>Speed of the shaft in RPM</td>
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<td>Nb</td>
<td>Number of balls used in ball bearing</td>
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<td>P</td>
<td>Maximum contact pressure on the bearing</td>
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<td>R</td>
<td>Equivalent radius of bearing</td>
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<td>Rb</td>
<td>Radius of the balls in the ball bearings</td>
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<td>S</td>
<td>Allowable strength of the Gear</td>
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<td>T</td>
<td>Torque acting on the shaft</td>
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<td>U</td>
<td>Pitchline velocity of ball bearing</td>
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<td>Maximum Load capacity of the bearing</td>
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*Maximum fillet which corner radius of bearing will clear.
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<th>OD D</th>
<th>Tolerance +.0000 to minus</th>
<th>Width B</th>
<th>Tolerance +.0000 to minus</th>
<th>* Milling drift</th>
<th>snap ring dimensions</th>
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<th>Basic Load Ratings</th>
<th>Limiting speed of Open Bearing (rpm)</th>
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